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CHARACTERISATION OF THE FLOODS IN THE DANUBE RIVER BASIN THROUGH FLOOD FREQUENCY AND SEASONALITY ANALYSIS

ANALIZA ZNAČILNOSTI POPLAV V POVODJU REKE DONAVE S POMOČJO VERJETNOSTNE ANALIZE IN ANALIZE SEZONSKOSTI

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Abstract

Floods are natural disasters that cause extreme economic damage and therefore have a significant impact on society. Understanding the spatial and temporal characteristics exhibited by floods is one of the crucial parts of effective flood management. The Danube River with its basin is an important region in Europe and floods have occurred in the Danube River basin throughout history. Flood frequency analysis (FFA) and seasonality analysis were performed in this study using the annual maximum discharge series data from 86 gauging stations in order to form a comprehensive characterisation of floods in the Danube River basin. The results of the study demonstrate that some noticeable clusters of stations can be identified based on the best-fitting distribution regarding FFA. Furthermore, the best-fitting distributions regarding FFA for the stations in the Danube River basin are generalized extreme values (GEV) and log Pearson type 3 (LP3) distributions as among 86 considered gauging stations, 76 stations have one of these two distributions among their two best fits. Moreover, seasonality analysis demonstrates that large floods in the Danube River basin mainly occur in the spring, and flood seasonality in the basin is highly clustered.

Keywords: Danube River basin, Floods, Flood Frequency Analysis (FFA), seasonality.

Izvleček

Poplave so ena izmed naravnih nesreč, ki povzročajo veliko gospodarsko škodo in zato močno vplivajo na družbo. Razumevanje prostorskih in časovnih značilnosti poplav je eden od ključnih dejavnikov učinkovitega upravljanja voda. Povodje reke Donave je pomembna regija v Evropi, poplave v povodju pa se pojavljajo že skozi celo zgodovino. V raziskavi smo izdelali celovito analizo poplav v povodju reke Donave, in sicer verjetnostne analize visokih vod in analize sezonskosti na podlagi maksimalnih letnih pretokov s 87 vodomernih postaj. Rezultati verjetnostnih analiz kažejo opazno grupiranje vodomernih postaj glede na

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najustreznejšo porazdelitveno funkcijo. Kot najustreznejši porazdelitveni funkciji za analize visokih vod na vodomernih postajah v povodju reke Donave sta se izkazali generalizirana porazdelitev ekstremnih vrednosti (GEV) in logaritemska Pearsonova 3 porazdelitev (LP3). Izmed 86 obravnavanih vodomernih postaj je bila kar za 76 postaj ena od omenjenih dveh funkcij med prvima dvema najustreznejšima. Analiza sezonskosti je pokazala, da se velike poplave v povodju reke Donave pojavljajo večinoma spomladi, sezonskost pojavljanja poplav pa je močno regijsko pogojena.

Ključne besede: povodje Donave, poplave, verjetnostna analiza visokih vod, sezonskost.

1. Introduction

Water resource management, planning, flood mapping, hydrologic/hydraulic designs, etc. all require a detailed knowledge about past flood events. Floods are defined in the Oxford dictionary as "an overflow of a large amount of water beyond its normal limits" (Stevenson, 2010). In more hydrologically sound language, Jarvis (1936) defined flood as "a relatively high flow as measured by either gauge height or discharge quantity", which is the definition used in this study.

Naturally occurring flood events vary in their frequency. In order to determine characteristics of the flood events and more specifically the design discharge rates and their frequency of occurrence (i.e. return period), probability theory methods can be applied. Return period is defined as "the average interval of time within which the given flood will be equalled or exceeded once" (ASCE, 1953, p. 1221). Flood frequency analysis (FFA) is one of the most commonly applied hydrologic procedures used to analyse the relationship between discharge and return period of the floods, which is unique for each individual gauging station (e.g. WMO, 1989; Bezak et al. 2014; Poduje et al., 2014; Vittal et al., 2015).

FFA can be performed using two types of sample definitions, namely annual maximum (AM) series and peaks-over-threshold (POT) series (e.g. Mitková and Onderka, 2010; Bezak et al., 2014; Šraj et al., 2015). The AM method relies on finding the maximum discharge rates for each year, which are then used as the annual maximum sample. This method's limitation is that individual large events, while still quite intense, may not actually be the yearly maximum and thus are not included in the sample. In order to compensate for this issue, the POT method can be applied; however, the POT

method also comes with its own limitations, such as setting the threshold, which could be very subjective and may not always help distinguish between different events (Bezak et al., 2014). In most practical cases the annual maximum discharge series (AM) is used to perform FFA (e.g. Šraj et al. 2012; Bezak et al. 2014; 2016; Bezak and Mikoš, 2014).

FFA is usually performed using statistical analysis in combination with different probability distribution functions. Several theoretical distribution functions are available for modelling exceedance magnitudes (e.g. WMO, 1989; Bezak et al. 2014). Generalized extreme values (GEV), Gumbel (G), Log normal (LN), Pearson type 3 (P3), Log Pearson type 3 (LP3), Generalized logistics (GL) are among the most commonly used ones. Some countries have guidelines suggesting which distribution to use for FFA. The four distributions that are most commonly used for FFA of AM series in individual countries are the GEV distribution (Australia, Austria, Cyprus, Germany, France, Italy, Lithuania, Slovakia, Spain), the Gumbel distribution (Finland, Greece), the GL (UK), and the log-Pearson III (USA, Australia, Lithuania, Poland, Slovenia) (Kobierska et al., 2018).

In order to fit the distribution parameters to the data, several methods can be applied, namely methods of moments, the method of L-moments, and maximum likelihood method (e.g. Hosking, 1990; Mahdi and Cenac, 2005; Grimaldi et al., 2011; Bezak et al., 2014). How the methods perform is influenced by the sample size and skewness of the data (Sankarasubramanian and Srinivasan, 1999); however, the method of L-moments has been found to be one of the most reliable methods in many researches because it is unbiased and more efficient than other methods (Lin and Vogel, 1993; Hosking and Wallis, 2005; Shahzad and Asghar, 2013; Bezak et al., 2014; Šimková and Picek, 2017).

The selection of the best-fitting distribution can be a challenge since more than one distribution may fit the data well (Salas et al., 2013). Various tests can be applied in order to check the adequacy of the tested distribution functions and find the best-fitting distribution. It is therefore recommended that several of them should be applied, since the goodness-of-fit tests are not necessarily unbiased and may favour one distribution over another (Kidson and Richards, 2005). NIST/SEMATECH (2010) provides several possibilities for goodnessof-fit tests such as the point plot correlation coefficient (PPCC) test, the Anderson-Darling (A-D) test, and the Kolmogorov-Smirnov (K-S) test. Other statistical tests, such as root-mean-square error (RMSE), mean absolute error (MAE), and the Akaike information criterion (AIC), are commonly used in statistics and have proven to be reliable methods of findings the best fits against given data (Iacobellis et al., 2010). Critical values for different tests can be find in the literature (e.g. (Chowdhury et al., 1991; Zeng et al., 2015; Bezak and Mikoš, 2014).

Seasonality analysis is an important part of flood characterisation. Several studies on the seasonal patterns of discharge data in Europe or in individual parts of the Danube River basin can be found in the literature (e.g. Parajka et al., 2009; 2010; Barbalič and Petraš, 2012; Hall and Blöschl, 2017; Blöschl et al., 2017; 2019). Parajka et al. (2009) analysed the precipitation and discharge data from stations in Slovakia and Austria. They argued that seasonality depended primarily on local characteristics. The study of Hall and Blöschl (2017) suggests that the geographical location of a station in Europe is a good indicator of its seasonal flood characteristics.

The frequency of floods in the Danube River basin increased in the last decades (e.g. major floods in

2002, 2005, 2006, 2009, 2010, 2013, and 2014), increasing the need for a more effective and harmonized regional and cross-border cooperation on flood protection (Šraj et al., 2019). Furthermore, reliable design discharge estimation is still a challenge for engineers and essential part of flood protection measures. Understanding the spatial and temporal characteristics of floods is one of the crucial parts of their effective management. Therefore, the main aims of the study are as follows: (i) a comprehensive analysis of discharge data series of the gauging stations in the Danube River basin, (ii) finding the most appropriate distribution functions for FFA in the Danube River basin and (iii) characterisation of the floods in the Danube River basin as regards seasonality.

2. Study area and data

2.1 The Danube River basin

The Danube is the second longest river in Europe after the Volga, with a length of 2850 km (Jones, 2007). It is a truly pan-European river, as it flows through 10 countries and its basin extends into 9 more. It is recognized as the world's most international basin. The countries in its basin are Germany, Austria, Slovakia, Hungary, Croatia, Serbia, Bulgaria, Romania, Moldova, Ukraine, Poland, Switzerland, Italy, Slovenia, Bosnia and Herzegovina, Montenegro, Macedonia, and Albania (Fig. 1). The Danube River basin extends over an area of 817,000 km² and is home to a population of 83 million people. As such, it is very important to local economies that water be provided for industry, agriculture, and municipalities, and, in some countries, it is important also for transportation. However, it is as a result subject to heavy impacts from such activities, with pollution being a growing concern (Jones, 2007; ICPDR, 2011).

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Figure 1: The Danube basin area.

Slika 1: Povodje reke Donave.

The Danube has many tributary rivers (327) and the four most important that contribute at least 10% of the Danube final average flow are the Drava, the Tisza, the Sava, and the Inn (ICPDR, 2011). Other important tributaries are also the Moravia, the Iskar, the Siret, and the Prut River.

The Danube River basin area has diverse geographic and climatic characteristics. An array of different climates is found in the basin area with continental climates influencing the majority of the basin and a minority being influenced by the Atlantic and Mediterranean climates (UNDP/GEF, 2010). Several mountain chains are also part of the basin. The Alps are located at the basin's northwest, the Carpathian Mountains in the east/centre, and the Dinarides constitute some of its south-west border. These are generally the wettest parts of the basin, with areas in the Alps that see rainfall as high as 3200 mm/year and between 750 and 2000 mm/year in the Carpathian Mountains. The mountains influence a large part of the basin with their rain shadows (UNDP/GEF, 2010). Several low-lying areas can be found as well. The lowlands of Moravia in the Czech Republic and the Tisza valley in Romania are fairly dry (up to 750 mm/year) and the Danube Delta is the driest (400 mm/year of precipitation) (UNDP/GEF, 2010).

The Danube River basin has experienced a number of significant floods over recent centuries with a total of 78 in the past 900 years, of which 23 occurred in the 18th century, before any significant flood protection was designed (ICPDR, 2011). Water marks have set record levels three times since 2002 and 5 significant floods have occurred in the past decade (ICPDR, 2011).

2.2 Data

The daily discharge time series used in the study has been obtained from the Institute of Hydrology at the Slovak Academy of Sciences as part of the UNESCO International Hydrological Program projects (Ninov and Brilly, 2017). The database consists of daily discharges from 86 measuring stations (21 of them being directly on the Danube River) in 11 countries in the Danube River basin (Table 1, Fig. 2). The number of available years for data varies from station to station with the maximum being for the station of Bratislava in Slovakia (D10) with 131 years of available data and the minimum being for a station Kozluk Jajce located in Bosnia (76) with 20 years of available data. The list of considered stations and their main characteristics is presented in Table 1. All data series were manually and visually examined (plotting values against time) for data errors (e.g. jumps in timing, large differences to surrounding stations), stability across time, and potential outliers to check their homogeneity. Additionally, it should be mentioned that there are several gaps in the data; a few sets of data have gaps during the Second World War (1939-1945), others have some years missing during the Balkan Wars (in the 1990s), and

some stations have random years missing inexplicably. It should also be noted that possible human influences on discharges were not investigated in this study; therefore, no station was excluded from analysis for this reason.

It is also important to notice that the considered stations and the rivers they are located on have different characteristics regarding their elevation, basin characteristics, climate etc. This means that the range of flow-rates and their averages vary greatly from station to station.



Figure 2: The considered water-gauging stations. Station codes are presented in Table 1. *Slika 2:* Obravnavane vodomerne postaje. Številke postaj so podane v preglednici 1.

Table 1: List of the considered gauging stations and their basic characteristics.

No.	Code	Country	Station Name	Lat	Lon	Basin Area [m³]	Height above see level [m]	# of years
1	1	GE	Inn-Oberaudorf	47.65	12.20	9712	464	107
2	2	GE	Inn-Passau-Ingling	48.65	13.45	26084	289	87
3	3	GE	Lech-Landsberg	48.04	10.88	2295	582	107
4	4	GE	Regen-Regenstauf -> replaced	49.22	12.17	2658	337	107
5	5	GE	Salzach-Burghausen	48.16	12.83	6649	352	107
6	6	GE	Issar-Plattling	48.77	12.88	8839	316	82
7	7	AT	Enns - Steyr, Ortskai	48.04	14.43	5915	284	55
8	8	AT	Traun - Ebensee	47.80	13.76	1257.6	422	55
9	9	CZ	Morava-kromeriz	49.30	17.40	7014	184	93
10	10	CZ	Morava - Straznice	48.93	17.30	9147	163	88

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11	11	CZ	Jihlava - Ivanice	49.08	16.41	2681	194	85
12	12	CZ	Svratka - Zidlochovice	49.04	16.62	3939	178	93
13	13	SK	Morava-Moravsky Jan	48.60	16.94	24129.3	146	85
14	14	SK	Bela-Podbanske	49.14	19.90	93.49	923	79
15	15	SK	Vah-Liptovsky Mikulas	49.09	19.61	1107.21	548	86
16	16	SK	Vah- (Trnovec - 1921-1962) Sala	48.16	17.88	11217.61	109	86
17	17	SK	Hron-Banska Bystrica	48.73	19.13	1766.48	334	76
18	18	SK	Hron-Brehy	48.41	18.65	3821.38	195	76
19	19	SK	Kysuca- Kysucke Nove Mesto	49.30	18.79	955.09	346	76
20	20	SK	Topla-Hanusovce nad Toplov	49.03	21.50	1050.05	160	76
21	21	SK	Krupinica-Plastovce	48.16	18.96	302.79	139	76
22	22	SK	Ipel-Holisa	48.30	19.74	685.67	172	76
23	23	SK	NITRA-NITRIANSKA STREDA	48.30	18.10	2093.71	158	78
24	24	HU	Raba-Arpas	47.51	17.40	6610	NA	54
25	25	HU	Tisza-Vasarosnameny	48.12	22.34	25100	102	126
26	26	HU	Tisza-Szolnok	47.17	20.19	73113	NA	88
27	27	HU	Tisza-Szeged	46.25	20.17	138408	74	87
28	28	HU	SZAMOS-CSENGER	47.83	22.68	15283	113	78
29	29	HU	MAROS-MAKO	46.22	20.48	30149	80	78
30	30	HU	SAJO-FELSOEZSOLCA	48.11	20.84	6440	107	117
31	35	SR	Tisza-Senta	45.93	20.10	141715	73	77
32	36	SR	Lim-Prijepolje	43.38	19.63	3160	442	83
33	37	SR	Drina-Bajina Basta	43.97	19.55	14797	211	82
34	38	SR	Sava-Sremska Mitrovica	44.98	19.62	87966	72	82
35	39 40	SR	Moravica-Arilje	43.75	20.12	832	322	45
36 37	40 41	SR SR	Ibar-Lopatnica Lakat Zapadna Morava - Jasika	43.63 43.62	20.57 21.30	7818 14721	225 139	60 49
37	41	SR	Juzna Morava-Mojsinje	43.62	21.30	14721	139	49 60
38 39	42	SR	Velika Morava-Ljubicevski most	43.03 44.58	21.48	37320	73	00 77
40	44	HR	-		18.20	37320	89	80
			Drava-Donji Miholjac	45.78				
41	45	HR	Kupa-Jamnicka kiselica	45.55	15.86	6895	101	53
42	46	HR	Sava-Zagreb	45.79	15.96	12450	112	81
43	47	HR	Orljava-Pleternica most	45.29	17.81	745	114	63
44	48	HR	Una-Kostajnica	45.22	16.55	8876	103	73
45	49	SL	Sava-Catez	45.89	15.61	10186.45	137	51
46	50	SL	Krka-Podboèje	45.87	15.47	2238.12	146	73
47	51	SL	Savinja Laško	46.15	15.23	1663.6	215	93
48	52	SL	Sava-Litija	46.06	14.82	4821.43	230	79
49	57	RO	Szamos-Satu Mare	47.80	22.88	15388	118	59
50	60	RO	Crisul Negru-Zerind	46.63	21.52	3702	87	58
51	62	RO	Maros-Arad	46.18	21.32	27280	118	57
52	64	UK	Siret-Storozhinec	48.15	25.72	672	356	52
53	65	UK	Prut-Chernivcy	48.32	25.92	6890	165	93
54	66	UK	Tisza-Rakhiv	48.07	24.22	1070	435	59
55	67	UK	Tisza-Vylok	48.10	22.83	9140	118	52
56	68	UK	Teresva-Ust-Chorna	48.33	23.93	572	524	58
50 57	69	UK	Rika-Mizhhirya	48.53	23.93 23.50	550	439	58 59
			•					
58	70	UK	Latorycya-Mucacheve	48.45	22.72	1360	123	59 40
59	71	UK	Latorycya-Chop	48.45	22.20	2870	105	49
60	72	UK	Uzh-Uzhhorod	48.62	22.30	1970	114	59
61	73	UK	Prut-Jaremcha	48.45	24.55	597	507	56
62	74	BA	Una-Kralje	44.84	15.85	NA	209	25

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63	75	BA	Sana-Sanski Most	44.77	16.68	2008	156	27
64	76	BA	Vrbas-Kozluk Jajce	44.37	17.29	3161	342	20
65	77	BA	Bosna-Maglaj	44.54	18.09	6619	150	26
66	D01	GE	Danube-Berg	48.27	9.73	4047	490	79
67	D02	GE	Danube-Ingolstadt Danube-Regensburg-	48.75	11.42	20001	360	84
68	D03	GE	Schwabelweis	49.02	12.14	35399	324	84
69	D04	GE	Danube-Pfelling	48.88	12.75	37687	308	82
70	D05	GE	Danube - Hofkirchen	48.68	13.12	47496	300	107
71	D06	GE	Danube-Achleiten	48.58	13.50	76653	288	107
72	D07	AT	Danube- Linz (ab 1979: Aschach) Danube - Stein-Krems /	48.31	14.30	79490	248	60
73	D08	AT	Kienstock	48.38	15.46	96028.4	194	104
74	D09	AT	Danube - Wien-Nußdorf	48.25	16.38	101700	156	107
75	D10	SK	Donau - Bratislava	48.14	17.11	131338	128	131
76	D11	HU	Danube-Nagymaros	47.78	18.95	183534	100	115
77	D12	HU	Danube-Mohács	46.00	18.67	209064	80	78
78	D13	SR	Danube-Bezdan	45.85	18.87	210250	81	76
79	D14	SR	Danube-Bogojevo	45.53	19.08	251593	77	77
80	D15	SR	Danube-Pancevo	44.87	20.64	525009	67	77
81	D16	SR	Danube-Veliko Gradiste	44.80	21.40	570375	62	77
82	D17	RO	Donau - Orsova	44.70	22.40	576232	44	106
83	D18	RO	Danube_Zimnicea	43.63	25.36	658400	16	107
84	D19	RO	Donau - Vadu Oii-Hirsova	44.68	27.92	709100	3	69
85	D20	RO	Danube Ceatal Izmail	45.22	28.72	807000	0.6	77
86	D21	RO	Danube - Reni	45.47	28.22	805700	4	90

3. Methods

3.1 Flood frequency analysis

In the study, the AM method was applied to define a sample as it is an objective method, independent of the subjective determination of a threshold, which would be different from station to station. Furthermore, some authors reported that the advantage of the POT method is noticeable mainly on smaller samples (Robson and Reed, 1999; Mitková and Onderka, 2010), which is not the case for most of the considered stations in the study. In order to define the AM samples for all considered stations of the Danube River basin, a code written in the R programming language was applied (R Core Team, 2018) using the daily discharge data series. The maximum daily discharge for each year, as well as the corresponding date of this discharge being filtered out. This was done using the package "hydroTSM" (Zambrano-Bigiarini, 2017b). It should be noted that not each annual maximum necessarily results in a flood; however, AM

sampling is nevertheless the most commonly used sample definition method in flood frequency analysis and flood characterisation (e.g. Bezak et al., 2014; Bezak et al., 2016; Blöschl et al., 2017; Blöschl et al., 2019).

The parameters of the considered distributions were estimated using the method of L-moments, which has been found as one of the most reliable methods in many studies (e.g. Hosking and Wallis, 2005; Bezak et al., 2014; Šimková and Picek, 2017). Additionally, L-moments are more robust and less sensitive to outliers than other methods (Hosking, 1990). Moreover, it was found to yield the best fit to the AM discharge sample for the Litija station on the Sava River, which is also a part of the Danube basin (Šraj et al., 2012). A detailed description of Lmoments calculation procedure can be found in Hosking (1990). In the study, L-moments were calculated using the "lmomco" package (Asquith, 2018) in R software (R Core Team, 2018). The package generates the L-moments up to an order of 5, along with their ratios. Log L-moments needed for certain distributions were also calculated.

In the next step of the study, several of the most commonly used distributions in univariate flood frequency analysis (FFA) were applied in order to find the best-fitting distribution, namely the General extreme value (GEV), Gumbel (G), Log normal (LN), Pearson type 3 (P3), Log Pearson type 3 (LP3), and Generalized logistics (GL). These distributions have been previously applied for FFA in many European studies. The LP3 and LN were tested on four stations in the Danube River basin by (Mitková and Onderka, 2010). The LP3 is also the distribution recommended by the USGS for the determination of flood frequencies (England et al., 2018). The Gumbel and GEV methods were applied for FFA on data from rivers in France (Kochanek et al., 2014), being a country right beside the basin area of the Danube River. Šraj et al. (2012) compared all mentioned distributions in their study using the data of the Litija station on the Sava River, which is also a part of the Danube River basin. Distribution functions and equations for distribution parameter estimation can be found in Šraj et al. (2012). Distributions were fit to the AM samples with a code in the R programming language (R Core Team, 2018) using several functions.

Traditional statistics such as root-mean-square-error (RMSE), mean-absolute-error (MAE), Akaike information criteria (AIC), as well as specific goodness-of-fit tests, such as the probability plot correlation coefficient (PPCC), Kolmogorov-Smirnov test (K-S), and Anderson-Darling test (A-D), were applied to find the best-fitting distribution in this study. Detailed descriptions and equations can be found in Šraj et al. (2012). Several of these tests can be applied using packages in R software, whereas others had to be programmed manually. MAE and RMSE tests could be found in the "hydroGOF" package (Zambrano-Bigiarini, 2017a) and the A-D test could be found in the package "ADGof" (Bellosta, 2011). The K-S test is part of the package "stats" (R Core Team, 2018). PPCC and AIC tests were programmed manually for purposes of this study (Morlot, 2018).

The best-fitting distributions are then ranked accordingly for each test. This was done by

assigning the highest value (6) to the best-fitting distribution and so on to the worst-fitting distribution assigned with the lowest value (1). Finding the best result depends on the test characteristics, since for MAE, RMSE, and AIC the lower the value of statistics means the better the fit. According to the PPCC test, better fits have values closest to 1. For the K-S and A-D tests, values closest to 0 are better. In the next step all values of ranks (from 1 to 6) were added together for each distribution of individual station, with the maximum being 36 (6 distributions * 6 tests). The highest score provides the best-fitting distribution. Using this ranking system, it was possible to find the bestfitting distribution for each station of the Danube River basin. Based on these results as well as the geographical coordinates of the stations, a map showing the best-fitting distribution for each individual station in the Danube River basin was created.

3.2 Seasonality

Seasonality analysis gives us important information about the time of flooding. It provides the most common season and dates for the occurrence of past floods, and allows us to predict the same parameters for the future floods (Bayliss and Jones, 1993; Burn, 1997).

In order to estimate the seasonality of the floods (AM) and their variability, several steps must be conducted. Seasonality can be graphically presented by a circular diagram (Burn, 1997; Bezak et al., 2016; Hall and Blöschl, 2017). The day of the year is transformed into an angle θ_i (for this purpose called angular date or Julian date J, which is the number of the day in the year between 0 and 365 (or 366 in a leap years)) by equation (1) (Bayliss and Jones, 1993):

$$\theta_i = \frac{J_i}{_{365}} * 2\pi \tag{1}$$

An average flood event, its average date of occurrence, and its season are also found for the AM series, as well as the variability between dates. The methodology used was proposed by Burn (1997) and defined by equations (2):

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} \cos\left(\theta_i\right) \tag{2}$$

$$\overline{y} = \frac{1}{n} \sum_{i=1}^{n} \sin(\theta_i)$$
$$\overline{\theta} = \tan^{-1}\left(\frac{\overline{y}}{\overline{x}}\right)$$

 $r = \sqrt{x^2 + y^2}$

where r represents the strength of seasonality. If r is near 1, the seasonality is strong and most of the considered events occurred at the same time of year. On the other hand, if r is closer to 0, the seasonality is not significant. Graphical presentation of the seasonality analysis was conducted using the 'circlize' package of the R programming language (Gu et al., 2014).

For the purpose of the study, the seasons were assumed to occur always at the same dates. Season start and end dates and corresponding Julian and angular dates are presented in Table 2.

 Table 2: Definition of the seasons in the study.

Preglednica 2: Opredelitev časovnih obdobij v raziskavi.

Season	Start and	Julian date	Angular
	end of the		date
	season		
Winter	Dec 22	356	-0.15
	Mar 20	79	1.36
Spring	Mar 21	80	1.37
	Jun 21	172	2.96
Summer	Jun 22	173	2.97
	Sep 22	265	4.56
Fall	Sep 23	266	4.57
	Dec 21	355	6.11

4. Results and discussion

4.1 Flood frequency analysis

The main aim of FFA was a comprehensive analysis of discharge data series from 86 gauging stations in the Danube River basin and finding the most appropriate distribution function for FFA in the Danube River basin. The best-fitting distribution function for each individual gauging station was found according to methodology described in the Methods section. The results of the analysis are presented in Fig 3, Fig. 4 and in Table 3.

The most common best-fitting distributions are as follows: GEV (22 stations), Pearson type 3 (20 stations), log Pearson type 3 (18), and GL (18 stations). On the other hand, Gumbel (3 stations) and LN (5 stations) were less commonly selected as the best-fitting distributions for the considered gauging stations in the Danube River basin (Table 3).

Table 3: No. of stations as regards the best-fittingdistribution.

Preglednica 3: Število postaj glede na najbolje prilegajočo se porazdelitev.

The best-fitting	Rank 1	Rank 2
distribution	(No. of	(No. of
	stations)	stations)
GL	18	8
GEV	22	20
Gumbel	3	3
LN	5	8
LP3	18	37
P3	20	10

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Figure 3: The best-fitting distribution for individual gauging station in the Danube River basin (Morlot, 2018).

Slika 3: Najbolje prilegajoča se porazdelitev za posamezno vodomerno postajo v povodju reke Donave (Morlot, 2018).



Figure 4: Gauging stations with Log Pearson type 3 or GEV among their two best-fitting distributions (Morlot, 2018).

Slika 4: Vodomerne postaje s porazdelitvijo Log Pearson 3 ali GEV, kot eno izmed dveh najbolje se prilegajočih (Morlot, 2018).

Statistical test results and scores according to the methodology described in Methods section are presented in Fig. 5. Detailed results for each individual station can be found in Morlot (2018). It should be mentioned that 13 of the considered stations have ties in the first place (two best-fitting distributions), with a score difference of zero. Furthermore, for the large majority of the considered stations (57 out of 86), the score difference between the first- and the second-best-fitting distribution is less than or equal to 4 (out of 36), indicating good performance for the best-fitting distributions.

Some noticeable station clusters can be identified in Fig. 3. However, no large grouping of stations is particularly obvious in general. For example, the downstream section of the Sava River and its tributaries seem to favour the Pearson type 3 distribution, whereas the upstream section of the Sava River favours GEV and GL distributions. Furthermore, five consecutive stations on the Danube River in Austria yield GL distribution as the best-fitting one, whereas five other stations on the Danube River between Serbia and Romania favour GEV as the best-fitting distribution.

Histogram of scores



Figure 5: Histogram of scores according to 6 statistical tests and the methodology described in section 3.

Slika 5: Histogram ocen 6 statističnih testov po metodologiji, opisani v razdelku 3.

It is worth emphasizing that among 86 considered stations 76 have either GEV or LP3 between their two best fits (Fig. 4). Given that many stations also have low scores and low differences between the scores according to our methodology, we can propose LP3 and GEV distributions as preferred distributions for FFA in the Danube River basin. The lower Danube basin seems to favour GEV distribution, whereas the upper Danube basin favours LP3 distribution and the middle part of the basin is a mix between both distributions. However, there are some exceptions (7 stations), where neither of two mentioned distributions gave the best fit (Fig. 4).

The study's findings are consistent with the results from previous studies. Šraj et al. (2012) investigated the influence of the choice of the method on the results of FFA for the Litija station on the Sava River (station 52 of this study) in Slovenia. They tested seven distributions and found LP3 distribution as the best-fitting one. Furthermore, GL distribution gave the worst fit. Their results are in accordance with findings of this study. There have also been some other studies on FFA for the stations in the Danube River basin; however, their findings could not be directly compared with the results of this study because of the limited number of the methods applied in their research. For example, Bačová-Mitková and Onderka (2010) analysed extreme hydrological events at five gauging stations on the Danube River, comparing the annual maximum and partial duration series. They applied only two different distributions to the AM data series, namely LN and LP3. However, they did not choose the best-fitting one, as they only provided the estimated flood quantiles using both considered distributions. In their next study, Bačová-Mitková and Halmova (2014) performed bivariate FFA for extreme hydrological events of the Danube River at the Bratislava gauging station, applying only the Gumbel distribution for the univariate analysis of peaks.

4.2 Seasonality

Based on the dates of flood peak occurrence, several factors defining seasonality, namely the most common flood season (the season with the greatest

number of AM occurrences), the season of the maximum flow-rate on the record, as well as the average flood season (the season of an average date of all AM), were investigated. Furthermore, the average date of the flood for each considered station using equation 2 ($\bar{\theta}$) as well as the variability of this date (*r*) was defined.

The number of occurrences of floods in different seasons (winter, spring, summer, autumn) for each individual measure of seasonality is presented in Table 4. The most common flood season in the Danube River basin is spring, followed by winter, summer, and then fall. Similar results are noticeable for the average flood season. For the season of the largest flood on the record, the number of occurrences was found to be different from other seasonality characteristics with the largest flood occurring most often in the spring, followed by summer, autumn, and winter. **Table 4:** Number of occurrences of floodsregarding different measures of seasonality.

Preglednica 4: Število pojavov poplav glede na različne opredelitve sezonskosti.

Season	Most common flood season	Average flood season	The season of the largest flood on the record
Winter	28	31	11
Spring	34 (+1)	36	33
Summer	14 (+1)	14	25
Autumn	9	5	17





Slika 6: Sezonskost poplav glede na najbolj pogosto obdobje poplav (z največjim številom pojavov visokovodnih konic) in povprečno poplavno obdobje (glede na povprečni datum pojava vseh visokovodnih konic) v povodju reke Donave (Morlot, 2018).



Figure 7: An example of analysis of the occurrence of the floods for the Danube River at Bratislava, where the red line represents the average Qmax at the average occurrence date and the purple line the largest Qmax on record.

Slika 7: Primer analize pojava poplav za reko Donavo v Bratislavi, kjer rdeča črta predstavlja povprečni Qmax na povprečni datum pojava in vijolična črta največji Qmax obravnavanega obdobja.



Figure 8: Seasonality and variability of annual flood dates in the Danube River basin (Morlot, 2018). *Slika 8:* Sezonskost in časovna spremenljivost letnih poplav v povodju reke Donave (Morlot, 2018).

Geographically, the flood season in the catchment appears to be highly clustered, as can be seen in Fig. 6. A grouping of average spring flooding is noticeable in the downstream areas of the Danube catchment area. Average summer flooding occurs mostly downstream of the northern slopes of the Alps. Average winter flooding seems to occur in clusters throughout the catchment area and characteristically occurs only on tributaries of the Danube River, except for a few cases very upstream on the Danube River (Germany). The average fall flooding occurs mostly in the Balkan region (Slovenia and Croatia). Furthermore, as noticeable from Fig. 6, most stations had the same average and the most common flood season.

Additionally, analysis of the date of the flood was conducted. An example of analysis for the station in Bratislava is presented in Fig. 7. Results for the entire Danube River basin show that plains with the Tisza River and the Alps region seem to be areas of high variability in seasonality (Fig. 8; Morlot, 2018). On the other hand, the upper part of the Danube River, as well as its tributaries such as the Sava and Drava rivers seem to have very small variability in flood occurrence dates (Morlot, 2018). Previous works on seasonality at the European scale (Hall and Blöschl, 2017) and national scales, such as Croatia (Barbalić and Petraš, 2012) or Slovenia (Bezak et al., 2016), yielded similar results regarding the average seasons and dates.

5. Conclusions

The main aim of the study was to characterize floods in the Danube River basin based on univariate flood frequency analysis (FFA) and seasonality.

FFA was conducted using six different distributions fitted to discharge data series from 86 gauging stations in the Danube River basin. The most common best-fitting distributions are GEV, P3, LP3, and GL. On the other hand, Gumbel and LN were less commonly selected as the best-fitting distributions for the considered gauging stations in the Danube River basin. Furthermore, the results of the study show that some noticeable clusters of stations can be identified based on the best-fitting distribution. For example, the downstream section of the Sava River and its tributaries favour P3 distribution, whereas the upstream section of the Sava River favours GEV and GL distributions. Furthermore, five consecutive stations on the Danube River in Austria favour GL distribution, whereas five stations on the Danube River between Serbia and Romania favour GEV distribution. Additional analysis demonstrated that among 86 considered stations 76 have either GEV or LP3 distribution between their two best fits. It should be noted that the results of the study have implications for flood risk management mainly in medium- and large-sized catchments of the Danube River basin, since FFA is based on a database of daily discharges.

Seasonality analysis demonstrates that the most common flood season in the Danube River basin is spring, followed by winter, summer, and fall. Similar results are obtained for the average flood season. On the other hand, the largest floods on record occur most often in the spring, followed by summer, autumn, and winter. Further analysis demonstrates that flood season in the catchment appear to be highly clustered.

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