

# APPLICABILITY OF COPULA FUNCTIONS IN HYDROLOGY

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Nejc Bezak



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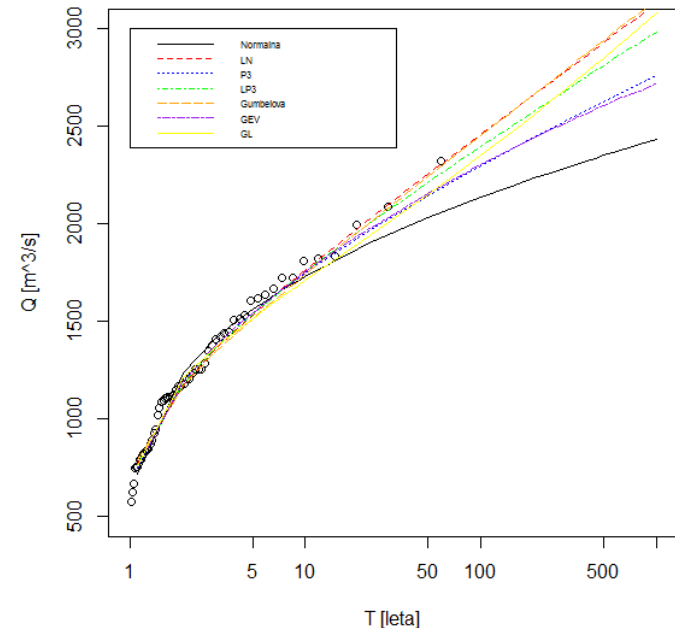
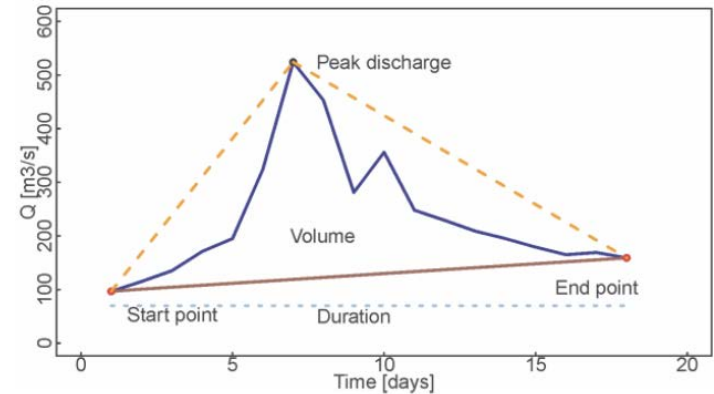
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• UNESCO Chair on  
• Water-related Disaster Risk Reduction  
• University of Ljubljana, Ljubljana, Slovenia

# COPULA FUNCTIONS?

- Hydrological processes are often multi-dimensional.
- Multivariate flood frequency approach can be used to simultaneously study two or more variables.
- Copula functions can be applied to perform the multivariate flood frequency analysis.

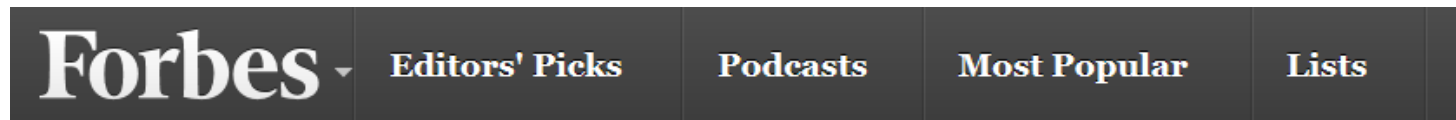


# COPULA FUNCTIONS

- Copula functions have been used for multivariate flood frequency analyses, modelling droughts, rainfall analyses, IDF curves determination, to check adequacy of dam spillway, for flood coincidence risk analyses, geostatistical interpolations and also several other hydrological and environmental applications (more than 350 papers from 2003).

The screenshot shows the website of the International Association of Hydrological Sciences (IAHS) and the International Commission on Statistical Hydrology (ICSH). The page title is 'International Association of Hydrological Sciences ICSH International Commission on Statistical Hydrology'. The navigation menu includes 'ICSH', 'Activities', 'Topics', 'Events', 'Forum', and 'Contacts'. The current page is 'Activities > ICSH References'. A dropdown menu is open under 'Copula Function', listing several topics: 'Regional Frequency Analysis', 'Non-stationarity', 'Entropy function in hydrology', 'Long memory and parametric models', and 'Artificial Intelligence Models'. The page content includes the heading 'References on Copula Function' and a note: 'IN THIS PAGE WILL BE REGULARLY UPDATED. This page is managed by Jing Li, University of...'. There is also a section titled 'REFERENCES ON COPULA FUNCTION'.

# COPULA FUNCTIONS



The Little Black Book of Billionaire Secrets



Cookies on Forbes



Susan Lee

5/08/2009 @ 12:01AM

## Formula From Hell

Every crisis has its evildoers. And the list associated with the current financial crisis is long, ranging from the famous Alan Greenspan to the merely infamous Richard Fuld.

But now the list has grown again with the addition of something called the Gaussian copula. At first, this evildoer excited scorn only on finance blogs, but lately it's making appearances in the [mainstream financial media](#).

HYDROLOGICAL PROCESSES

*Hydrol. Process.* 29, 225–238 (2015)

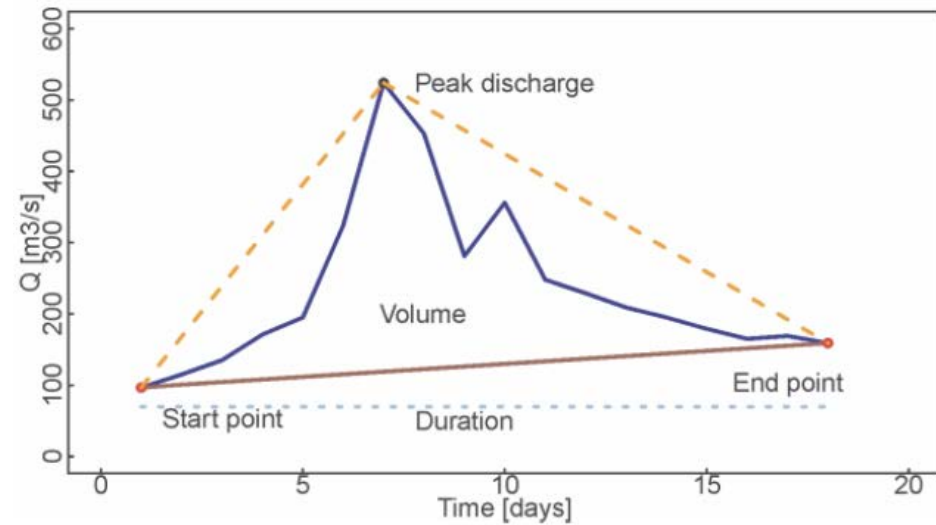
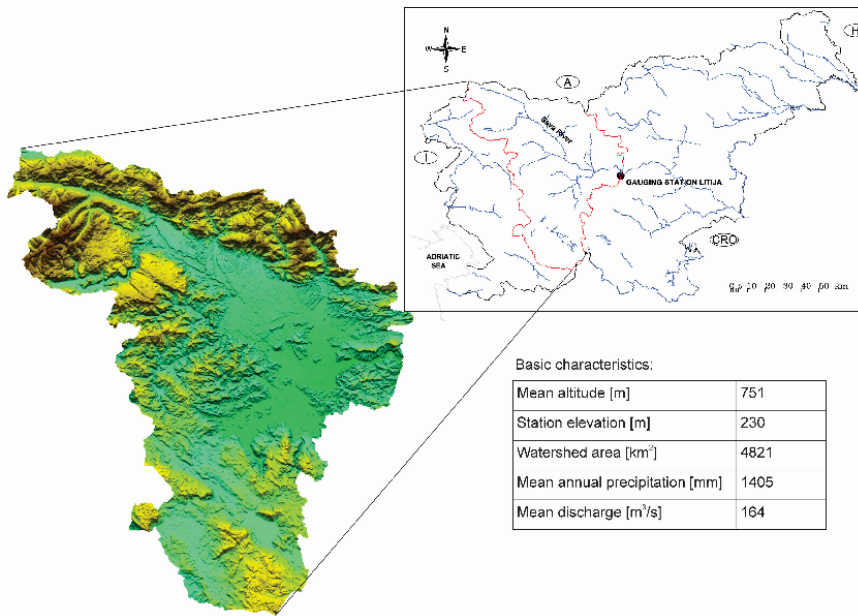
Published online 30 January 2014 in Wiley Online Library

(wileyonlinelibrary.com) DOI: 10.1002/hyp.10145

## Bivariate flood frequency analysis using the copula function: a case study of the Litija station on the Sava River

Mojca Sraj,\* Nejc Bezak and Mitja Brilly

*Faculty of Civil and Geodetic Engineering, University of Ljubljana, Jamova 2, SI-1000, Ljubljana, Slovenia*

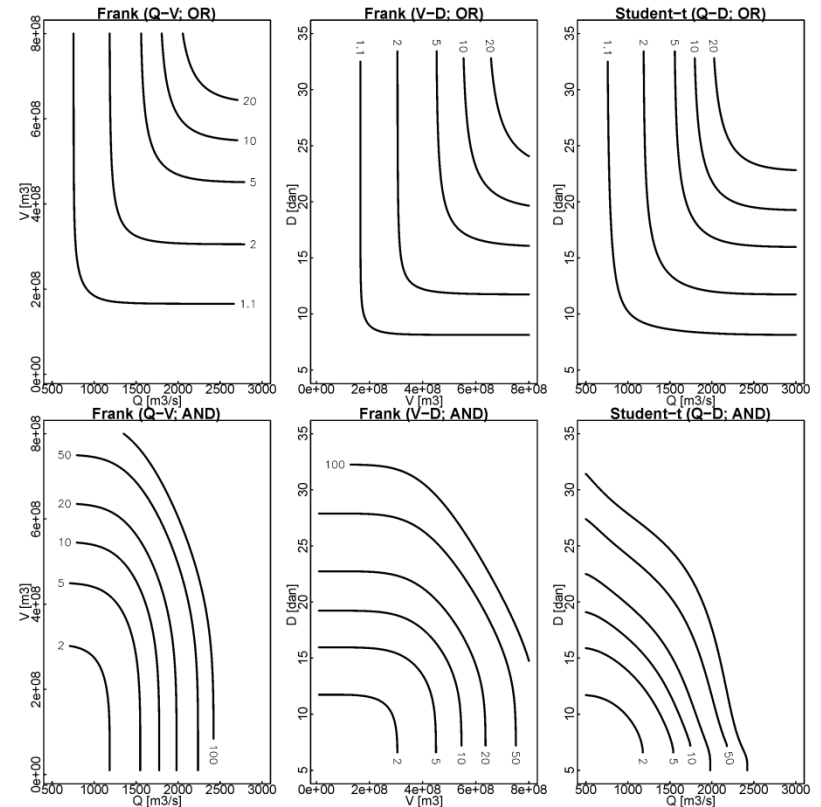
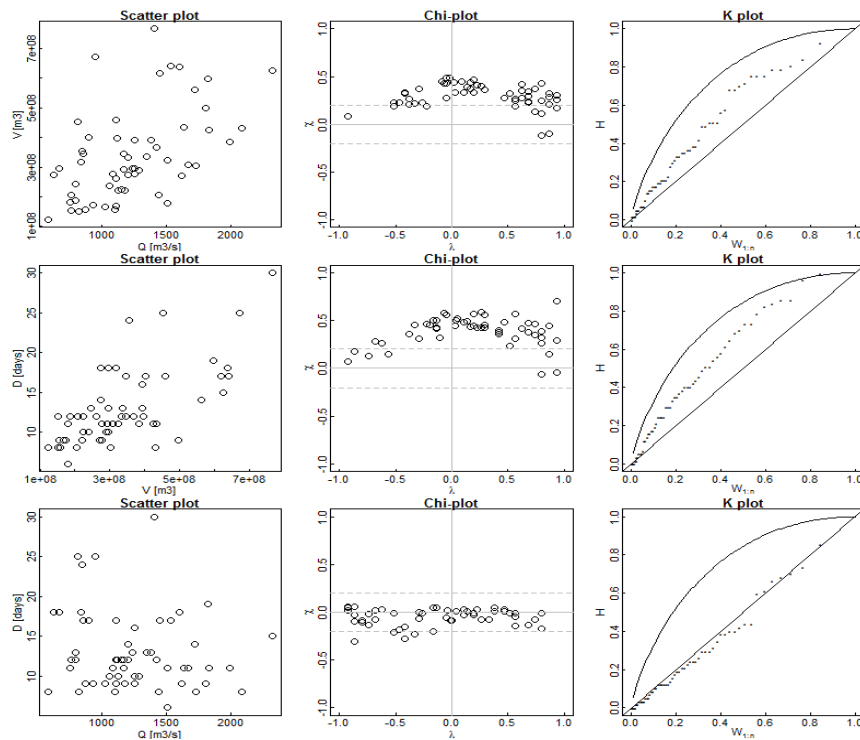


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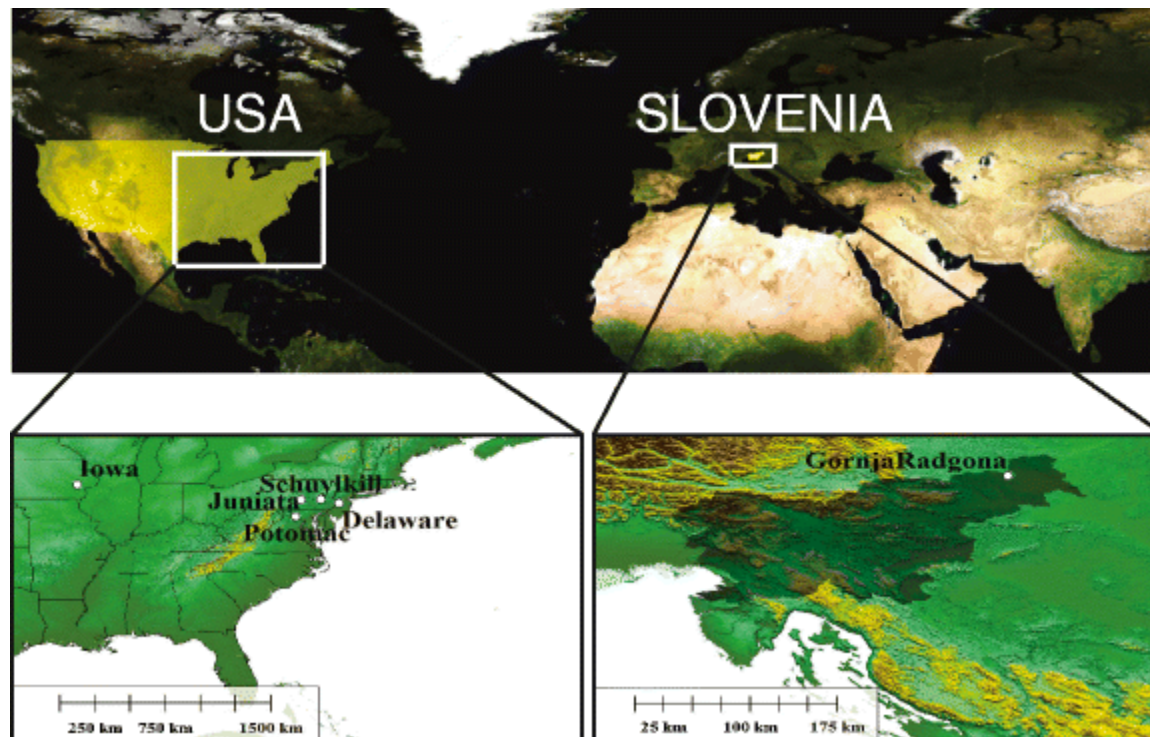


Water Resour Manage (2014) 28:2195–2212  
DOI 10.1007/s11269-014-0606-2

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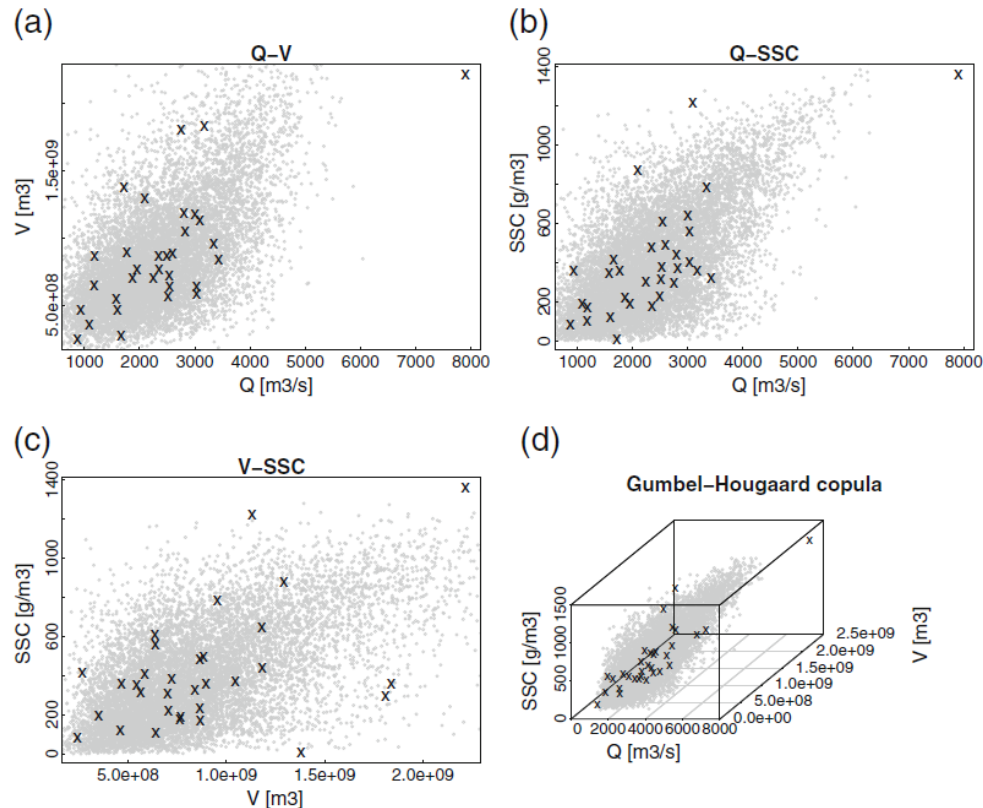
## Trivariate Frequency Analyses of Peak Discharge, Hydrograph Volume and Suspended Sediment Concentration Data Using Copulas

Nejc Bezak • Matjaž Mikoš • Mojca Šraj

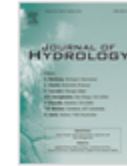


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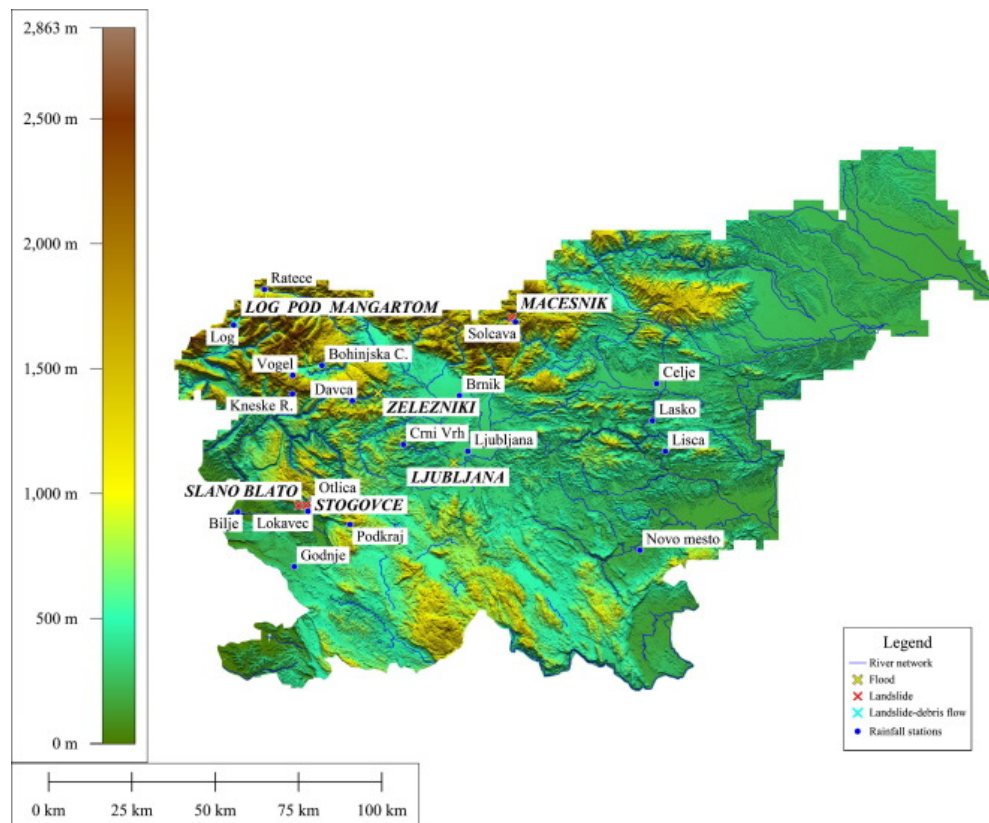


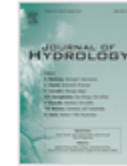




## Copula-based IDF curves and empirical rainfall thresholds for flash floods and rainfall-induced landslides

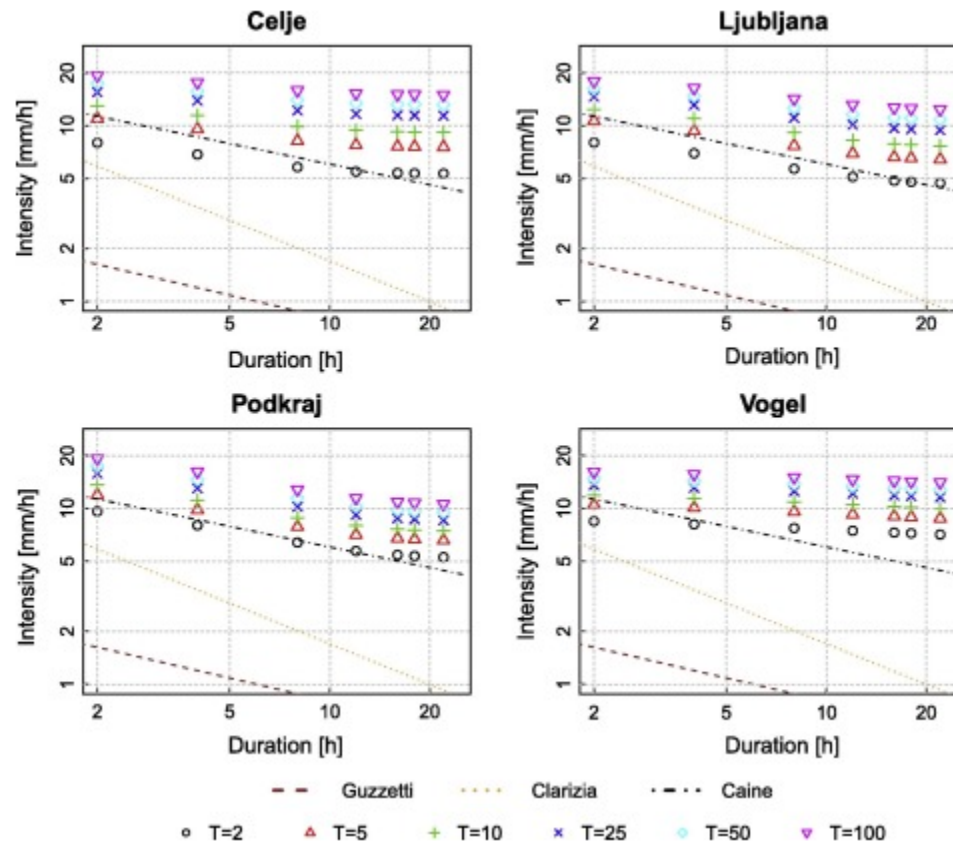
Nejc Bezak  , Mojca Šraj  , Matjaž Mikoš  





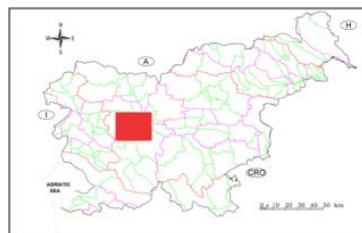
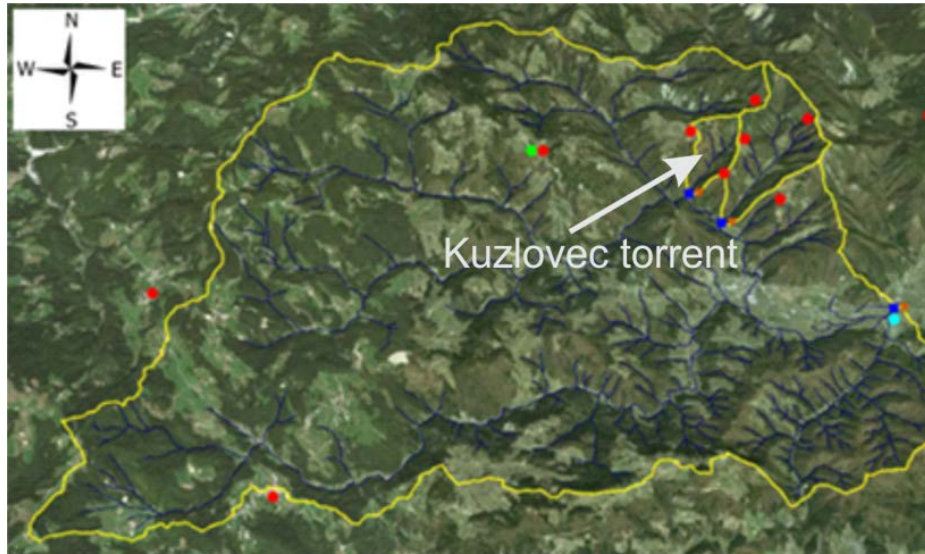
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# ONGOING WORK

- Estimation of suspended sediment loads using copula functions



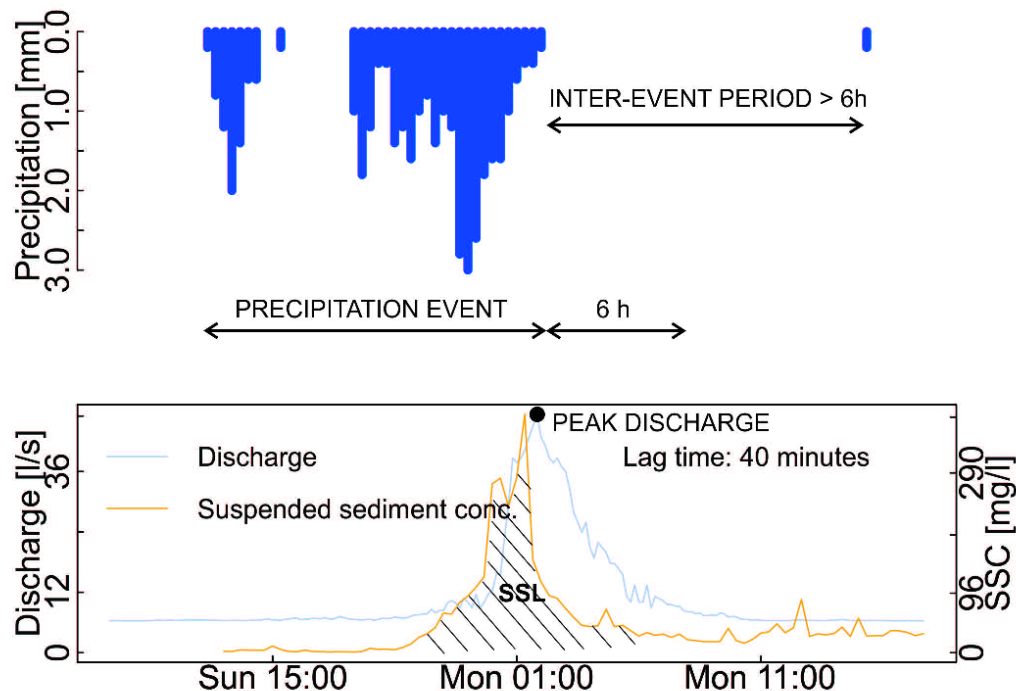
- Meteorological station
  - Rainfall station
  - Disdrometer
  - Water level gauge
  - ▼ Turbidity meas.
- 0 1 2 km



# ONGOING WORK

- Estimation of suspended sediment loads using copula functions

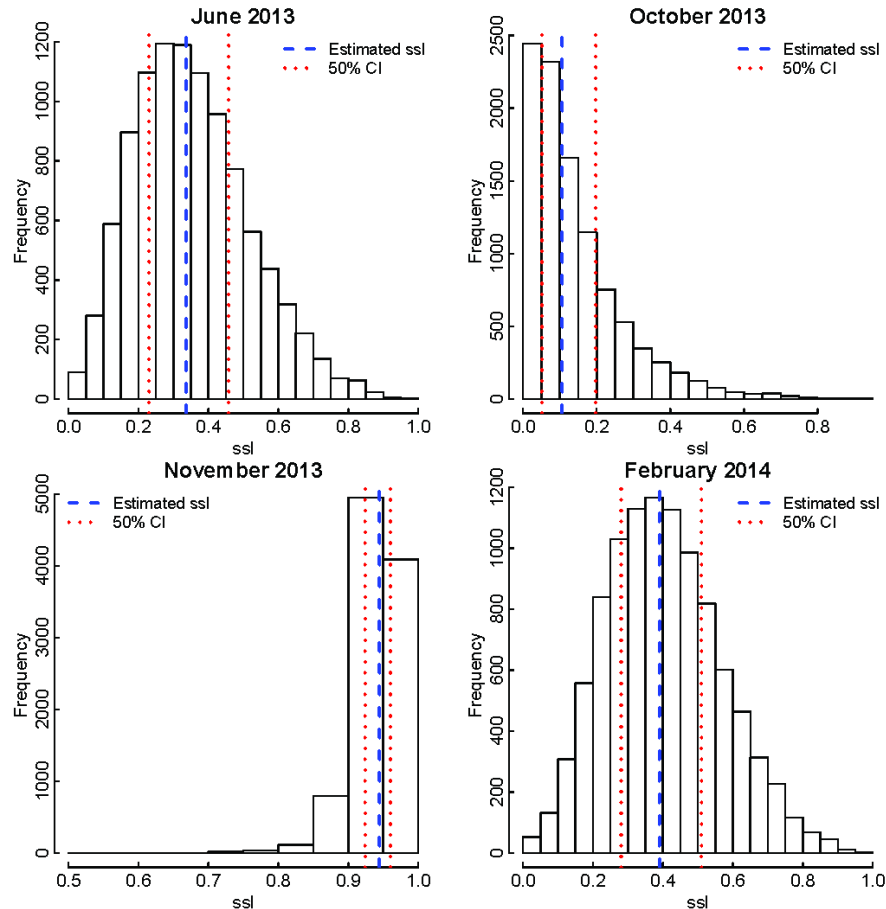
$$C_{\theta}(u_3|u_1, u_2) := P(U_3 \leq u_3 | U_1 = u_1, U_2 = u_2) = \frac{\partial^2 C_{\theta}(u_1, u_2, u_3)}{\partial u_1 \partial u_2} / \frac{\partial^2 C_{\theta}(u_1, u_2)}{\partial u_1 \partial u_2}$$





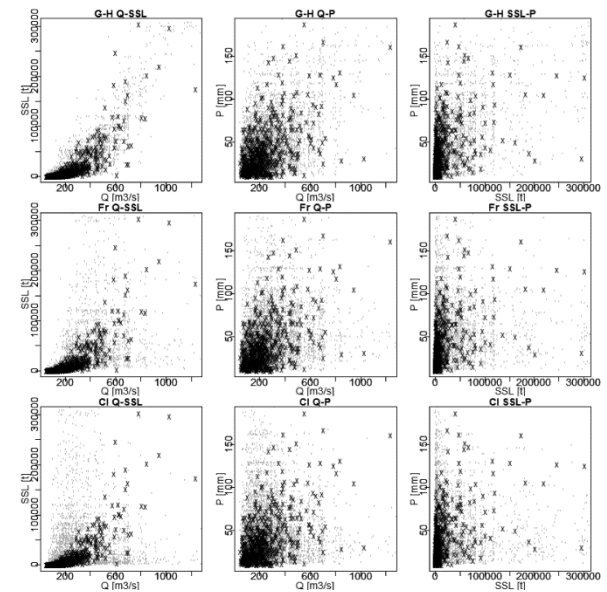
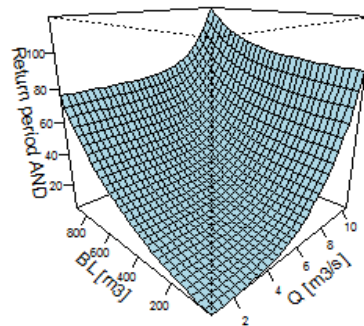
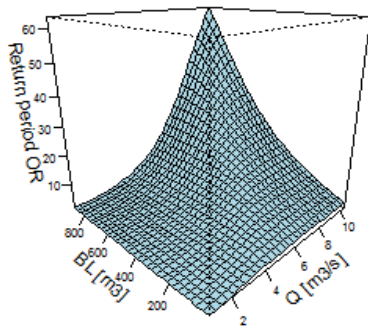
# ONGOING WORK

- Estimation of suspended sediment loads using copula functions



# MAIN CONCLUSIONS

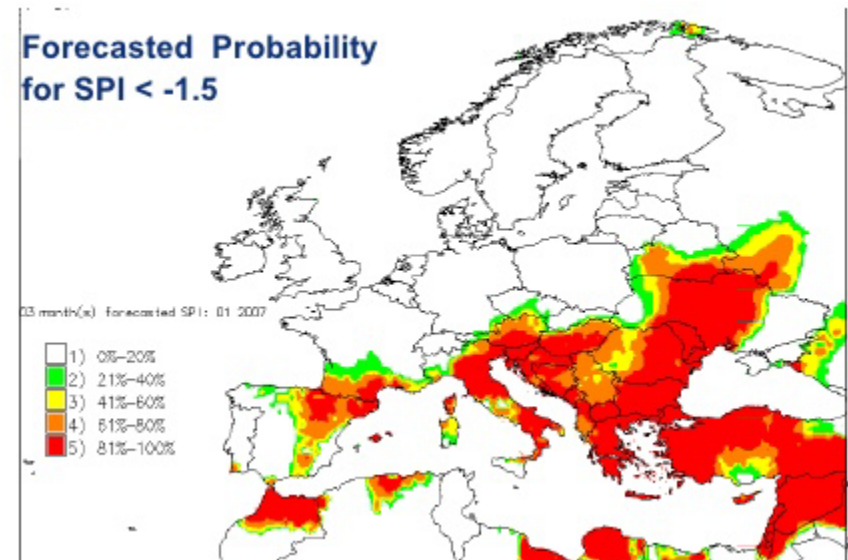
- Copula functions were applied to several hydrological applications in Slovenia.
- Copula frequency analysis can be used as an alternative to the univariate frequency approach.





# NEW IDEAS

- Multivariate regional flood frequency analysis using copula functions.
- Implementation of probabilistic approach in the shallow landslide early-warning systems using copula functions.
- Your suggestions??



# THANK YOU FOR YOUR ATTENTION



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